

The Anti-Cloak

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Abstract

A kind of transformation media, which we shall call the “anti-cloak”, is proposed to partially defeat the cloaking effect of the invisibility cloak. An object with an outer shell of “anti-cloak” is visible to the outside if it is coated with the invisible cloak. Fourier-Bessel analysis confirms this finding by showing that external electromagnetic wave can penetrate into the interior of the invisibility cloak with the help of the anti-cloak.

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The ideas of transformation optics and cloaking [1-4] have attracted keen interest both in theory [1-18] and in experiment [19]. The cloaking effect has been proved using different methods, such as ray tracing [8], full wave simulation employing finite element methods [9] and the finite difference time domain methods [10], and the Mie scattering models [11,12]. In particular, Ruan et al. [11] employed Mie scattering models to confirm that a cylindrical cloak with the ideal material parameters is indeed a perfect invisibility cloak using Fourier-Bessel analysis. Similar approaches [12] were used to confirm the perfect cloaking effect of the spherical cloak. However, essentially all the aforementioned examples that demonstrated the perfect cloaking effect did not consider embedded objects with material anisotropy inside the cloak. In this paper, we show that the perfect cloaking effect can be defeated by adding another kind of transformation media inside the cloak (i.e., the anisotropy inside the cloak is considered, see in Fig. 1 schematically). We shall construct an example which demonstrates that an object with an outer shell of a specific form of negative index anisotropic material cannot be made entirely invisible by the transformation media cloak.

Starting from the mapping [6, 14] (see in Fig. 2(a)), $r = \frac{b-r_0}{b-a}(r' - b) + b$, the required parameters for a partial cylindrical cloak (transverse electric (TE) mode is considered here) are obtained as follow,

$$\mu_r = \frac{r' - a_1}{r'}, \mu_\theta = \frac{r'}{r' - a_1}, \varepsilon_z = \left(\frac{b - r_0}{b - a}\right)^2 \frac{r' - a_1}{r'}, \quad (1)$$

where $a_1 = \frac{a-r_0}{b-r_0}b$. This partial cloak can reduce the total scattering cross section of a perfect electrical conductor (PEC) cylinder from its radius $r' = a$ to an equivalent PEC cylinder whose radius is $r = r_0$. In the limit as r_0 goes to zero, the partial cloak becomes perfect [1].

Now let us add another coordinate transformation inside the cloak ($c < r' < a$) as depicted in Fig. 2(a), $r = \frac{d-r_0}{c-a}(r' - c) + d$. The corresponding material parameters are then,

$$\mu_r = \frac{r' - a_2}{r'}, \mu_\theta = \frac{r'}{r' - a_2}, \varepsilon_z = \left(\frac{d - r_0}{c - a}\right)^2 \frac{r' - a_2}{r'}, \quad (2)$$

where $a_2 = \frac{ad-cr_0}{d-r_0}$. We note that these values are negative. We call this kind of transformation media the "anti-cloak" as we shall see that they cancel partially the effect of an invisibility cloak.

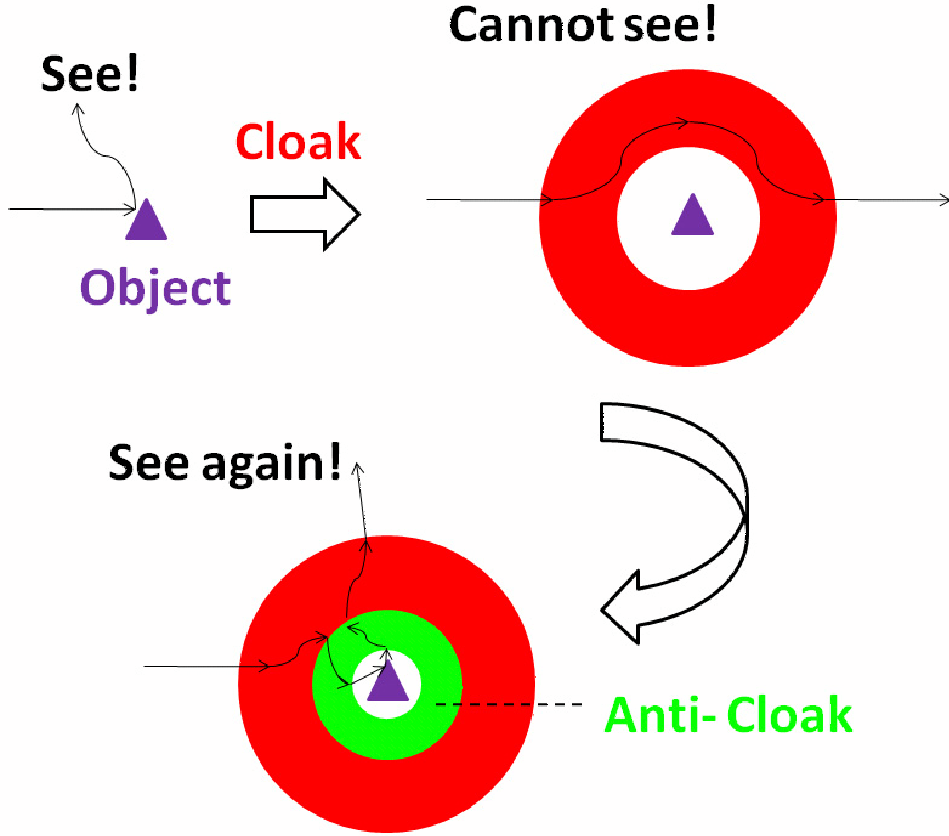


FIG. 1: (Color online) The schematic figure to illustrate the cloaking effect and anti-cloaking effect.

In the same spirit of the partial cloak, when a PEC cylinder with a radius $r' = c$ is coated with the anti-cloak in direct contact with the partial cloak, the total scattering cross section will be changed into that of an equivalent PEC cylinder whose radius is $r = d$. We note that there are no PEC boundary between the cloak and the anti-cloak (at $r' = a$), they are in direct contact.

Doing the same Fourier-Bessel analysis in [11], we can obtain the electric fields in each region,

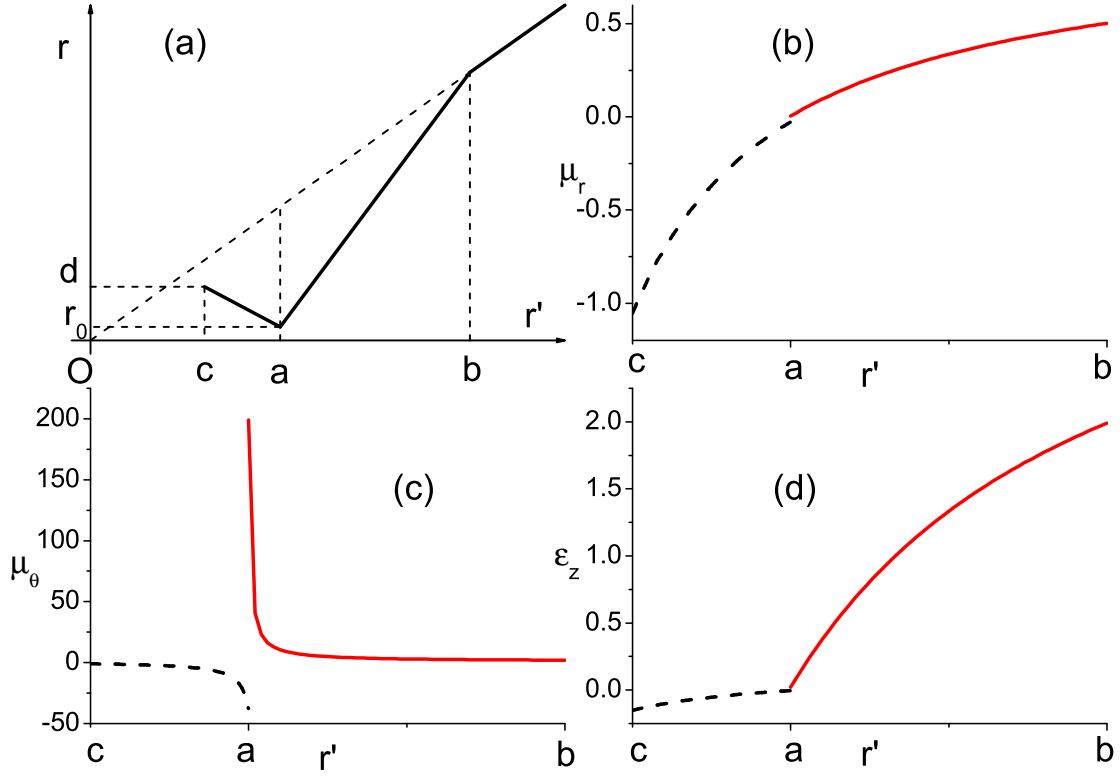


FIG. 2: (Color online) (a) The coordinate transformation of the cloak ($a < r' < b$) and anti-cloak ($c < r' < a$). (b) μ_r for cloak and anti-cloak. (c) μ_θ for cloak and anti-cloak. (d) ε_z for cloak and anti-cloak. The red solid lines in b-d denote the parameters of cloak, while the black dashed lines in b-d denote the parameters of anti-cloak.

$$\begin{aligned}
 (b \leq r) : E_z &= \sum_l \alpha_l^{in} J_l(k_0 r) \exp(il\theta) \\
 &\quad + \alpha_l^{sc} H_l(k_0 r) \exp(il\theta), \\
 (a \leq r < b) : E_z &= \sum_l \alpha_l^1 J_l(k_1(r - a_1)) \exp(il\theta) \\
 &\quad + \alpha_l^2 H_l(k_1(r - a_1)) \exp(il\theta), \\
 (c \leq r < a) : E_z &= \sum_l \alpha_l^3 J_l(k_2(r - a_2)) \exp(il\theta) \\
 &\quad + \alpha_l^4 H_l(k_2(r - a_2)) \exp(il\theta).
 \end{aligned} \tag{3}$$

where $J_l \backslash H_l$ are the l -order Bessel \backslash Hankel function of the 1st kind, k_0 is the wave vector of

the light in vacuum, $k_1 = \frac{b-r_0}{b-a}k_0$, $k_2 = \frac{d-r_0}{c-a}k_0$, α_l^{in} and α_l^{sc} are the incident and scattering coefficients outside the cloak, $\alpha_l^i (i = 1, 2, 3, 4)$ are the expansion coefficients for the field in the cloak and anti-cloak. The primes are dropped for aesthetic reasons from here. From the continuous boundary conditions (at $r = b$ and $r = a$) and the PEC boundary ($E_z = 0$ at $r = c$), we can obtain that,

$$\begin{aligned}\alpha_l^1 &= \alpha_l^3 = \alpha_l^{in}, \\ \alpha_l^2 &= \alpha_l^4 = \alpha_l^{sc}, \\ \alpha_l^{sc} &= -\frac{J_l(k_0 d)}{H_l(k_0 d)} \alpha_l^{in}.\end{aligned}\tag{4}$$

This result confirms that the PEC cylinder with its radius $r = c$ coated with the anti-cloak and cloak is equivalent to a PEC cylinder with its radius $r = d$ in the view of outside world.

To demonstrate the properties of the anti-cloak, we set $a = 0.1m$, $b = 0.2m$, $c = 0.05m$, $d = 0.02m$, $r_0 = 0.001m$. We plot the parameters of the cloak and anti-cloak at different radial positions in Fig. 2(b)-(d). All the parameters of anti-cloak are negative because of the negative slope of the coordinate transformation. A plane wave is incident from left to right with the frequency $2GHz$. In Fig. 3(a), we plot the scattering pattern of a PEC cylinder with a radius r_0 . The tiny PEC cylinder causes little scattering for the incoming plane wave which can be treated as almost invisible. In Fig. 3(b), we plot the scattering pattern of a PEC cylinder with a radius a coated by a partial cloak. The outer radius of the cloak is b . We see that the partial cloak reduces substantially the scattering of the PEC cylinder with its radius a when we compare Fig. 3(a) and Fig. 3(b). When r_0 is made as small as we like, the scattering becomes vanishing small. In Fig. 3(c), we plot the scattering pattern of a PEC cylinder with a radius c coated by an anti-cloak and a partial cloak [20]. The anti-cloak is located in $c < r < a$, the cloak locates in $a < r < b$. Without the anti-cloak, the wave basically goes around the shielded region, but if the anti-cloak is in contact with the cloak, EM wave from outside can go into the anti-cloak to interact with the object inside. The scattering of the cloak is enlarged again to that of an equivalent PEC cylinder whose radius is d . We plot the scattering pattern of the equivalent PEC cylinder in Fig. 3(d). When $c = d$, the anti-cloak together with the partial cloak becomes invisible, that means one can directly see the PEC cylinders with radius $r = c$, and the anti-cloak cancels out the effect

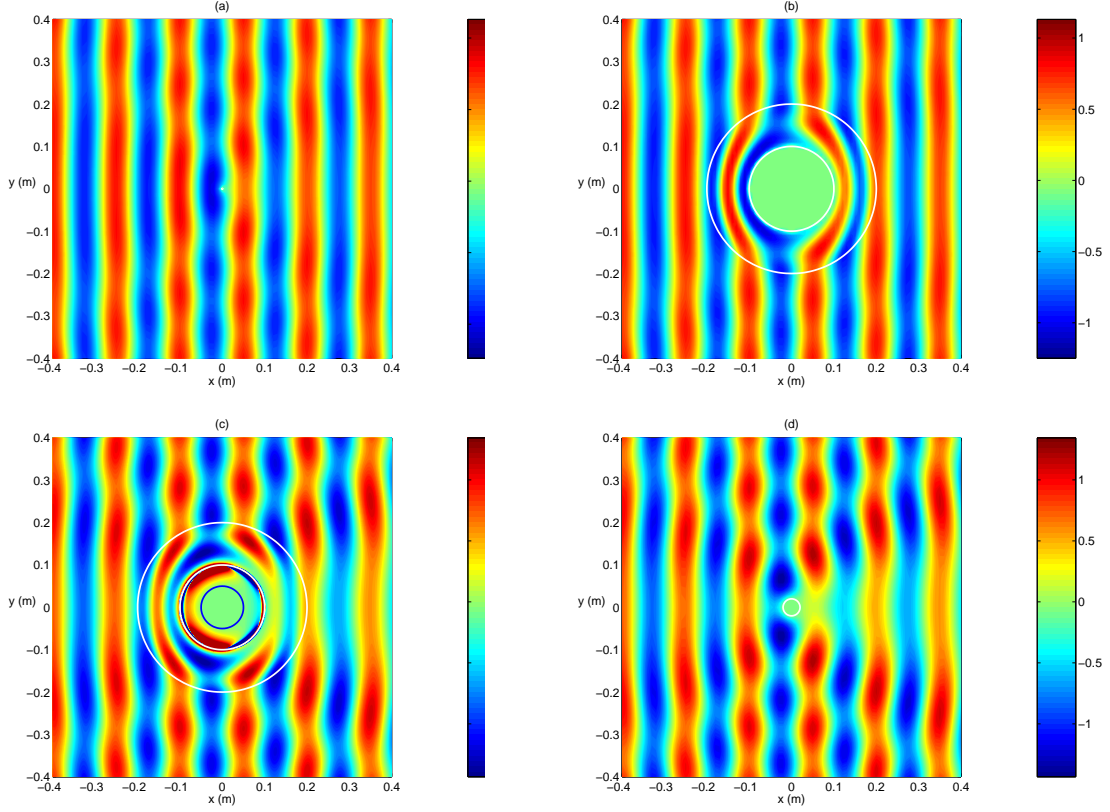


FIG. 3: (Color online) The electric field distribution for, (a) a tiny PEC cylinder with a radius r_0 (outlined by the white point); (b) a PEC cylinder with a radius a wearing a partial cloak (the outer radius is b , the inner and outer boundaries of the cloak are outlined by the white solid lines); (c) a PEC cylinder with a radius c wearing an anti-cloak and the same partial cloak as in (b) (the inner and outer boundaries of the cloak are outlined by the white solid lines while the inner boundary of the anti-cloak at $r = c$ is outlined by the blue solid line); (d) a PEC cylinder with a radius d (outlined by the white solid line).

of the partial cloak completely. For aesthetic reasons, if the electric field is larger than the maximum value in color bar in Fig. 3(c), we have replaced this overvalued field with the maximum value when plotting Fig. 3(c). If the electric field is smaller than the minimum value, we have replaced this overvalued field with the minimum value when plotting Fig. 3(c).

Due to the continuous coordinate transformation at $r = a$, the impedances are matched

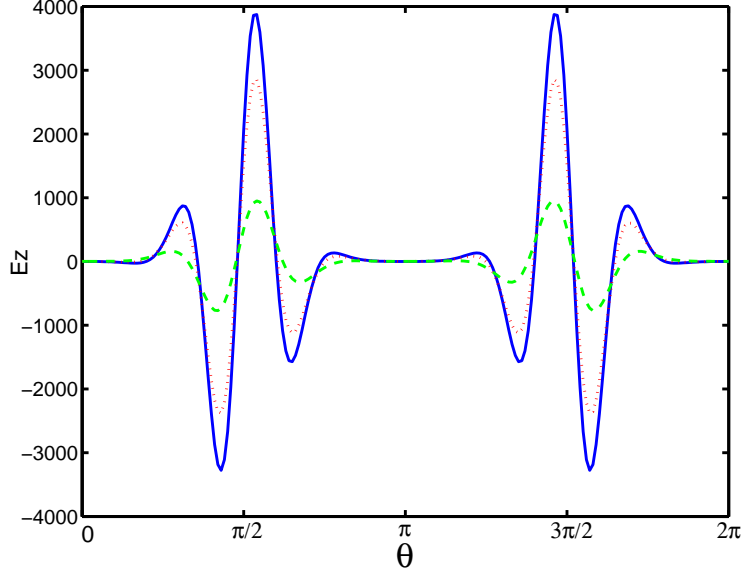


FIG. 4: (Color online) The electric field distribution at $r = a - 0.1r_0$ (red dashed line), $r = a$ (solid blue line) and $r = a + 0.1r_0$ (green dotted line).

at this touching boundary of the cloak and anti-cloak. The electric field is very large at this touching boundary. To show this property, we plot the electric field for different angles at fix radii near $r = a$ in Fig. 4. Three fixed radii are chosen, $r = a - 0.1r_0$, $r = a$ and $r = a + 0.1r_0$. We find that the electric field near $r = a$ is very large.

Analytically, one can obtain the electric field at $r = a$ as follow,

$$\begin{aligned} E_z &= \sum_l \alpha_l^{in} J_l(k_0 r_0) \exp(il\theta) + \alpha_l^{sc} H_l(k_0 r_0) \exp(il\theta) \\ &= \sum_l \alpha_l^{in} \exp(il\theta) [J_l(k_0 r_0) - \frac{J_l(k_0 d)}{H_l(k_0 d)} H_l(k_0 r_0)]. \end{aligned} \quad (5)$$

The term $H_l(k_0 r_0)$ becomes very large when r_0 is small, that is why we obtain large electric field above.

Since we can make r_0 as small as we like, we reach the conclusion that an almost perfect cloak can be defeated by an anti-cloak. In other words, the transformation media cloak is not a panacea as there exists some objects that it cannot hide. In the limit that r_0 is exactly zero, the situation requires further mathematical analysis due to the singularity properties of the anti-cloak and cloak ($H_l(k_0 r_0)$ diverges when r_0 goes to zero). From a physical standpoint, we may argue as follows. Near the inner boundary of the invisibility cloak, μ_r goes to zero

and μ_θ goes to infinity and they are positive, while near the outer boundary of the anti cloak, μ_r goes to zero and μ_θ goes to infinity from the negative side. The positive singular values have to come from an in-phase resonance while the negative infinity comes from out-of-phase resonance. If we put them in contact, the system response is canceled out, and the cloaking effect is weakened or even destroyed. The surface mode resonance at $r = a$ is excited and contributes to the large electric field. In addition, if the losses are considered, the electric field will become finite for r_0 is exactly zero. The cylindrical anti-cloak concept could be extended to three dimensions.

In conclusion, we find that the invisible cloak cannot hide the enclosed domain if the inside domain has a shell of anti-cloak. The properties are demonstrated by using the Fourier-Bessel analysis and finite-element full wave simulations. The anti-cloak region is an anisotropic negative refractive shell that is impedance matched to the cloak outside, which has a positive refractive index. It is known that [21] a negative refractive index medium “cancels” the space of a positive index medium that has the same impedance. So, a heuristic way of understanding the operation of an anti-cloak is that it annihilates the functionality of the interior part of the invisibility cloak, and effectively shifts the enclosed PEC region outwards to make contact with the outer part of the cloaking shell that is not “canceled”. This leads to a finite cross section.

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